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Quantum theory of the radiative interaction of charged particles

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Abstract. The presence of an external electromagnetic wave leads to an important change on the Coulomb interaction between two quantum charged particles and gives rise to long-range radiative forces. These effects result in the modification of Rutherford scattering.

1. Introduction

Recently it was found that the induced oscillations of the small particles under the action of the external fields result in the appearance of the long-range radiative forces [1-5]. These time-averaged forces are proportional to the square of the field amplitude and inversely proportional to the distance between the particles. The radiative forces acting between gas bubbles and between the solid corpuscles in a compressible liquid in a sound field were considered in [1-4]. The radiative interaction of charged particles in an external electromagnetic wave was investigated in [5]. The distance dependence of these forces is the same in all cases [1-5]. The small particles are considered to be classical in these papers. From the standpoint of the classical theory the radiative forces are caused by the secondary radiation of the particles. The external fields result in the induced oscillations of the particles. The oscillating particles radiate the secondary waves which create the radiative forces. Such possible interdisciplinary transfer between acoustics and classical electrodynamics is based on the formal analogy of the corresponding equations. It is not new in physics. To cite, for example, the articles [6-8]. It is also clear that the term particle itself has not the same significance in fluid mechanics and in electromagnetic theory (recently many works have investigated extended models of the electron [9-14]).

The purpose of our paper is to show that the quantum theory leads to analogous results. We shall obtain this for the example of quantum charges placed in the field of a plane monochromatic electromagnetic wave. The interactions between them are activated by the exchange of photons.

2. The interaction of the quantum charges in an electromagnetic wave

In the following we shall use the analytical solution of Schrödinger equation for the relativistic lone electron in a plane electromagnetic wave which was obtained by Volkov

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[15] ($\hbar = c = 1$)

$$\psi_p = \left(1 + \frac{e}{2(kp)} \hat{k}\hat{A} \cos(kx)\right) \frac{U(p)}{\sqrt{2p_0}} e^{iS(x)} \tag{1}$$

where

$$S(x) = -(px) - \frac{e(pA)}{(kp)} \sin(kx) + \frac{e^2A^2}{8(kp)} \sin(2kx). \tag{2}$$

Here $k = (w, \boldsymbol{k})$ and $A = (0, \boldsymbol{A} \cos(kx))$ are the wave 4-vector and the 4-vector of the external electromagnetic wave respectively (we suppose that $(kA) = -(\boldsymbol{k} \cdot \boldsymbol{A}) = 0$); $x = (t, \boldsymbol{r})$ is the 4-vector of the electron; to simplify the formulae the indexing is omitted in expressions (1) and (2) and the scalar product of vectors a and b is $(ab) = a_0b_0 - \boldsymbol{a} \cdot \boldsymbol{b}$; $U(p)$ is a constant bispinor. The equation for $U(p)$ is of the form

$$\left(\hat{p} + \frac{e^2A^2\hat{k}}{4(kp)} - m\right) U(p) = 0. \tag{3}$$

The square of the energy-momentum vector is

$$p^2 = m_{\text{eff}}^2 = m^2 + \tfrac{1}{2}e^2\boldsymbol{A}^2 \tag{4}$$

where m_{eff} is the effective mass of the electron [15]. We shall suppose that $\bar{U}U = 2m$. Then the condition of normalization for the functions (1) is

$$\frac{1}{(2\pi)^3} \int \psi_{p'}^*(x)\psi_p(x)\, d^3\boldsymbol{r} = \frac{1}{(2\pi)^3} \int \bar{\psi}_{p'}(x)\gamma^0\psi_p(x)\, d^3\boldsymbol{r} = \delta^3(\boldsymbol{p} - \boldsymbol{p}'). \tag{5}$$

This condition is of exactly the same type as that for free plane waves.

For the current density one obtains

$$j^\mu = \bar{\psi}_p\gamma^\mu\psi_p = \frac{1}{p_0}\left(p^\mu - eA^\mu - \frac{ek^\mu}{kp}(\boldsymbol{p} \cdot \boldsymbol{A}) \cos kx + \frac{e^2\boldsymbol{A}^2}{4(kp)} k^\mu \cos 2kx\right).$$

The time average $\overline{j^\mu} = p^\mu / p_0$.

Thus the solutions (1) are plane waves modulated by an external electromagnetic wave.

In order to describe the interaction of the charges in an electromagnetic wave we shall consider the Feynman diagram which is shown in figure 1.

This diagram corresponds to the exchange of a virtual photon between the particles 1 and 2. We suppose that these Fermi particles are different. The corresponding matrix element is [15]

$$S = ie_1e_2 \int d^4x\, d^4x'\, D_{\mu\nu}(x - x')\bar{\psi}_{p_1'}(x)\gamma^\mu\psi_{p_1}(x)\bar{\psi}_{p_2'}(x')\gamma^\nu\psi_{p_2}(x') \tag{6}$$

where

$$D^{\mu\nu}(x - x') = \frac{4\pi g^{\mu\nu}}{(2\pi)^4} \int d^4q \frac{e^{-iq(x-x')}}{q^2 + i\varepsilon} \tag{7}$$

is the photon propagator.

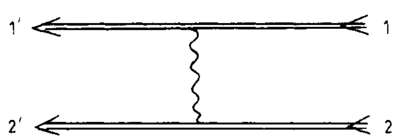

Figure 1. The elastic scattering of charged particles in the presence of an external electromagnetic wave.

Let us use the solutions (1) as the wavefunctions in (6) and consider the non-relativistic limit. Then the scalar products of the type (kp) are equal to $kp \approx \omega m_{\mathrm{eff}}$.

The matrix element takes the form

$$S=\frac{4\pi \mathrm{i} e_1 e_2}{(2\pi)^4}\int \mathrm{d}^4x\,\mathrm{d}^4x'\,\mathrm{d}^4q\,\frac{K(kx,kx',p_1,p_1',p_2,p_2')}{q^2+\mathrm{i}\varepsilon}$$
$$\times\exp[-\mathrm{i}q(x-x')-\mathrm{i}(p_1-p_1')x-\mathrm{i}(p_2-p_2')x'] \tag{8}$$

where

$$K(kx,kx',p_1,p_1',p_2,p_2')$$
$$=\left[1-\frac{\boldsymbol{A}^2}{4}\left(\frac{e_1^2}{m_{1_{\mathrm{eff}}}^2}+\frac{e_2^2}{m_{2_{\mathrm{eff}}}^2}\right)+\frac{\boldsymbol{A}^2}{2}\left(\frac{e_1}{m_{1_{\mathrm{eff}}}}\cos kx-\frac{e_2}{m_{2_{\mathrm{eff}}}}\cos kx'\right)^2\right]$$
$$\times\exp\left(\frac{\mathrm{i}e_1\boldsymbol{A}\cdot(\boldsymbol{p}_1-\boldsymbol{p}_1')\sin kx}{\omega m_{1_{\mathrm{eff}}}}+\frac{\mathrm{i}e_2\boldsymbol{A}\cdot(\boldsymbol{p}_2-\boldsymbol{p}_2')\sin kx'}{\omega m_{2_{\mathrm{eff}}}}\right). \tag{9}$$

Let us expand the function K in a Fourier series. For example one of terms in (9) can be represented as

$$\exp\left(-\frac{\mathrm{i}e_1(\boldsymbol{A}\cdot\boldsymbol{p})}{\omega m_{1_{\mathrm{eff}}}}\sin kx\right)=a_0+a_1\sin kx+a_2\sin 2kx+\ldots \tag{10}$$

where $\boldsymbol{p}=\boldsymbol{p}_1'-\boldsymbol{p}_1$.

The main member is [16]

$$a_0=\frac{1}{2\pi}\int_0^{2\pi}\exp\left(-\frac{\mathrm{i}e_1(\boldsymbol{A}\cdot\boldsymbol{p})}{\omega m_{1_{\mathrm{eff}}}}\sin\varphi\right)\mathrm{d}\varphi=J_0\left(\frac{e_1(\boldsymbol{A}\cdot\boldsymbol{p})}{\omega m_{1_{\mathrm{eff}}}}\right) \tag{11}$$

where $J_0(x)$ is the Bessel function. It follows from (11) that a_0 is the time average of the exponent (10).

We shall extract from the function K that part which is the function of the difference $(x-x')$. This result can be obtained by means of the mathematical operation

$$K_0(k(x-x'),p_1,p_1',p_2,p_2')=\frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\varphi\,K(kx+\varphi,kx'+\varphi,p_1,p_1',p_2,p_2'). \tag{12}$$

Let us calculate the expression (12) with an accuracy to the second order in the field amplitude to illustrate the operation. Then

$$K_0\approx 1+\frac{1}{4}\left(\boldsymbol{A}^2-\frac{(\boldsymbol{A}\cdot\boldsymbol{p})^2}{\omega^2}\right)\left(\frac{e_1^2}{m_1^2}+\frac{e_2^2}{m_2^2}-\frac{2e_1e_2}{m_1m_2}\cos k(x-x')\right). \tag{13}$$

After the change of variables $X=(x+x')/2$. $\xi=x-x'$ and calculation of matrix element (8) with the modified function (13), one obtains

$$S=-\mathrm{i}(2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')V(\boldsymbol{p}) \tag{14}$$

where

$$V(\boldsymbol{p})=\frac{4\pi e_1e_2}{\boldsymbol{p}^2}\left[1+\frac{1}{4}\left(\frac{e_1^2}{m_1^2}+\frac{e_2^2}{m_2^2}\right)\left(\boldsymbol{A}^2-\frac{(\boldsymbol{A}\cdot\boldsymbol{p})^2}{\omega^2}\right)\right]$$
$$+\frac{\pi e_1^2e_2^2}{m_1m_2}\left(\boldsymbol{A}^2-\frac{(\boldsymbol{A}\cdot\boldsymbol{p})^2}{\omega^2}\right)\left(\frac{1}{\omega^2-(\boldsymbol{p}-\boldsymbol{k})^2}+\frac{1}{\omega^2-(\boldsymbol{p}+\boldsymbol{k})^2}\right). \tag{15}$$

The function $V(\boldsymbol{p})$ can be treated as the Fourier image of the static potential of the two-particle interaction [15, 17]. If we go over to the usual space coordinates then

$$V(\boldsymbol{r})=\frac{e_1e_2}{r}+\frac{e_1e_2}{4}\left(\frac{e_1^2}{m_1^2}+\frac{e_2^2}{m_2^2}\right)A_\alpha A_\beta\left(\delta_{\alpha\beta}+\frac{1}{\omega^2}\frac{\partial^2}{\partial x_\alpha\partial x_\beta}\right)\frac{1}{r}$$
$$+\frac{e_1^2e_2^2}{2m_1m_2}\cos(\boldsymbol{k}\cdot\boldsymbol{r})A_\alpha A_\beta\left(\delta_{\alpha\beta}+\frac{1}{\omega^2}\frac{\partial^2}{\partial x_\alpha\partial x_\beta}\right)\frac{\cos kr}{r}. \tag{16}$$

It can be seen from (16) that the presence of the electromagnetic wave leads to the important change in the Coulomb interaction between the charged particles and results in the appearance of the long-range forces (last term in (16)) which decrease inversely proportional to the distance between the particles. We can say that the expression (16) is independent of the Dirac constant $\hbar$. Hence, the indicated corrections to the Coulomb interaction are of a classical character. This conclusion is in agreement with [1-5].

These results can be produced by other means. Let us consider the usual Lagrangian of quantum electrodynamics, including the classical electromagnetic wave

$$\mathscr{L}_{\rm int}=e\int\bar{\psi}(x)\gamma^\mu\psi(x)(A_\mu(x)+A_\mu^{\rm (cl)}(x))\,\mathrm{d}^3\boldsymbol{r}.$$

In order to take into account the influence of the external field we shall study the interaction with an accuracy to e^4. Then the Feynman diagrams which are shown in figure 2 result in the such a correction of the Coulomb potential as the expression (15).

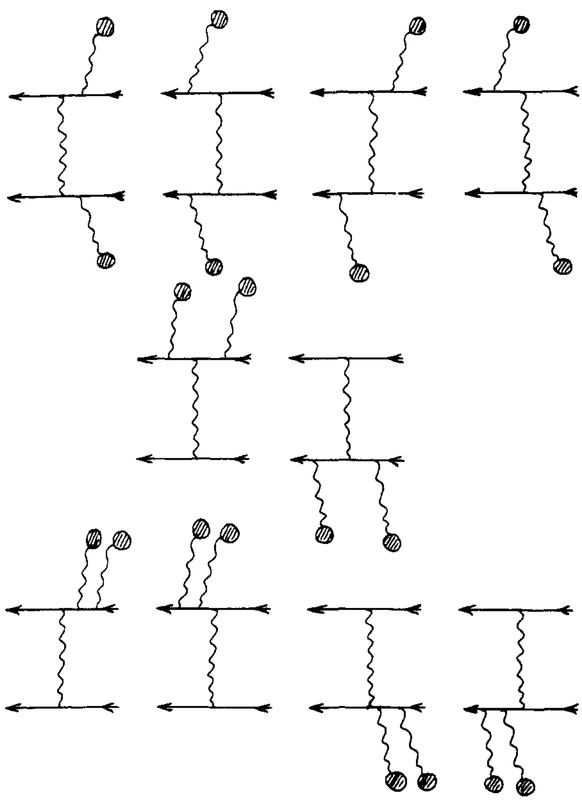

Figure 2. The Feynman diagrams of order e^4 which result in the modification of the Coulomb potential.

These diagrams correspond to the processes which describe the successive absorption and emission (or otherwise) of an external field quantum by the system of two electrons. Thus the combined energy-momentum vector of the electrons is preserved. These processes are presented by the factor $\delta^4(p_1+p_2-p_1'-p_2')$ in matrix element (8). As far as other contributions in the S-matrix are concerned (for example the terms of order e^3 are important in scattering processes also) they lead to the appearance of the factors $\delta^4(p_1+p_2-p_1'-p_2'\pm k)$, $\delta^4(p_1+p_2-p_1'-p_2'\pm 2k)\ldots$. These processes correspond to the absorption or emission of external field quantums by the system of two electrons. Therefore the combined energy-momentum vector of the electrons is not preserved.

In short we take an interest only in the processes which lead to the preservation of the combined energy-momentum of the two particles, i.e. we are looking at the problem only in the elastic channel.

Even though we are dealing with terms of the order e^4 in this paper, we exclude two-photon exchange process between the two charges [18]. This process requires special consideration.

After simple but unwieldy transformations the accurate expression for the two-particle potential (which is defined from calculation of matrix element (8) with the time-averaged function k_0) can be written as follows:

$$V(\boldsymbol{r})=\frac{e_1e_2}{2}\int_0^{2\pi}\frac{\mathrm{d}\varphi}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\tau\frac{[\delta(\tau-|\boldsymbol{\rho}(\tau,\boldsymbol{r},\varphi)|)+\delta(\tau+|\boldsymbol{\rho}(\tau,\boldsymbol{r},\varphi)|)]}{|\boldsymbol{\rho}(\tau,\boldsymbol{r},\varphi)|}$$
$$\times\left(1-\frac{e_1^2\boldsymbol{A}^2}{4m_{1_{\mathrm{eff}}}^2}-\frac{e_2^2\boldsymbol{A}^2}{4m_{2_{\mathrm{eff}}}^2}+\frac{\boldsymbol{A}^2}{2}\cos^2\varphi R^2(\tau,\boldsymbol{r})\right) \tag{17}$$

where

$$\boldsymbol{\rho}(\tau,\boldsymbol{r},\varphi)=\boldsymbol{r}-\frac{\boldsymbol{A}}{\omega}R(\tau,\boldsymbol{r})\sin\varphi$$

$$R(\tau,\boldsymbol{r})=\left(\frac{e_1^2}{m_{1_{\mathrm{eff}}}^2}+\frac{e_2^2}{m_{2_{\mathrm{eff}}}^2}-\frac{2e_1e_2}{m_{1_{\mathrm{eff}}}m_{2_{\mathrm{eff}}}}\cos(\omega\tau-\boldsymbol{k}\cdot\boldsymbol{r})\right)^{1/2}.$$

The resulting expression is very complicated. That is why we shall consider only one extreme $m_2\to\infty$. This case corresponds for example to the hydrogen-like atom in a plane electromagnetic wave. In this limit

$$R(\tau,\boldsymbol{r})\approx e_1/m_{1_{\mathrm{eff}}}.$$

We suppose that

$$e_1^2\boldsymbol{A}^2/m_1^2\ll 1. \tag{18}$$

Then the potential takes the form

$$V(\boldsymbol{r})=\frac{e_1e_2}{2\pi}\int_0^{2\pi}\frac{\mathrm{d}\varphi}{|\boldsymbol{r}-(e_1\boldsymbol{A}/m_1\omega)\sin\varphi|} \tag{19}$$

i.e. transforms to the Kramers–Henneberger potential (see, for example, [19] and references therein).

3. Rutherford scattering in the presence of an electromagnetic wave

Let us calculate the elastic scattering of an electron by the nucleus in the presence of an external wave. The interaction of an electron with the nucleus is determined by the effective potential (19). If we use the integral representation of the Bessel function (11) then in the first approximation of perturbation theory for the differential cross-section one obtains

$$d\sigma = d\sigma_R J_0^2\left(\frac{e_1(\boldsymbol{p}\cdot\boldsymbol{E})}{m_1\omega^2}\right) = \frac{e_1^2 e_2^2 m_1^2}{4|\boldsymbol{p}_1|^4 \sin^4(\Theta/2)} J_0^2\left(\frac{e_1(\boldsymbol{p}\cdot\boldsymbol{E})}{m_1\omega^2}\right) \tag{20}$$

where $\boldsymbol{p} = \boldsymbol{p}_1' - \boldsymbol{p}_1$, Θ is the scattering angle, $\boldsymbol{E} = \boldsymbol{A}\omega$ is the amplitude of the electric intensity vector, $d\sigma_R$ is the usual differential cross-section (when $\boldsymbol{E} = 0$) which is given by the Rutherford formula [17]. The modification is the appearance of the factor $J_0^2(Z)$, where $Z = e_1(\boldsymbol{p}\cdot\boldsymbol{E})/(m_1\omega^2)$. Let us suppose that $\boldsymbol{p}_1 \| \boldsymbol{E}$ and $(\boldsymbol{p}_1\cdot\boldsymbol{k}) = 0$. Then the argument of Bessel function is

$$Z = 2Z_0 \sin^2\frac{\Theta}{2} = 2|e_1|\frac{|\boldsymbol{p}_1||\boldsymbol{E}|}{m_1\omega^2}\sin^2\frac{\Theta}{2}.$$

For electrons with energy of about 1 keV, at wavelength $\lambda = 1$ mu and $|\boldsymbol{E}| = 10^6$ v cm^{-1} one obtains the value $Z_0 \simeq 1$. As for the parameter $|e_1|\cdot|\boldsymbol{A}|/m_1$, it is equal to 3×10^{-5}.

The appearance of the factor $J_0^2(Z)$ leads to the next effect. The elastic-scattering cross-section of electrons can be equal to zero at some scattering angles. These angles correspond to zeros of the Bessel function and may be verified by experimental measurements.

4. Conclusions

The presence of an external wave results in important changes of interaction between the quantum particles. These corrections are analogous to the corresponding changes arising in classical acoustics and electrodynamics.

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